1 Proof of returns to scale of the production function

A total product $TP$ of the production of given crime is the total damage $D$ dealt (i.e. $TP = D$). Consider a production function of total damage which is a function of number of offenders:

$$TP = D = f(n)$$

Every offender in the group is paid the average product $AP(n) = AD(n)$ of the production function defined by equation (1). We denote this share as the damage per person $D_{pp}$. Clearly:

$$AP(n) = AD(n) = \frac{TP(n)}{n} = \frac{D(n)}{n} = D_{pp}(n)$$

The marginal product $MP$ of the production function is in this context called marginal damage $MD$ (i.e. $MP=MD$). It is defined simply as:

$$MD(n) = MP(n) = D(n) - D(n - 1)$$

Equation 3 corresponds to the equation (2.3) on page 8 in the thesis. Clearly, if $MD(n) > 0$ then the total damage is increasing and if $MD(n) = 0$ total damage is constant and if $MD(n) < 0$ total damage is decreasing. This of course does not say anything about the returns to scale of the production function.

*This material is made for the defense of the thesis "Returns from Cooperation in Criminal Activity: Estimating Crime Production Function and Returns to Scale"; the defense takes place on Monday, January 29th at 11 a.m. at the University of Economics in Prague*
Now, since every offender is paid equal average product which we call
damage per person $D_{pp}$, we can define the marginal damage per person $MD_{pp}$
as a change between average product of $n$ and $n - 1$ participants.

\[ MD_{pp}(n) = AP(n) - AP(n-1) = D_{pp}(n) - D_{pp}(n-1) = \frac{D(n)}{n} - \frac{D(n-1)}{n-1} \]  

Equation (4) corresponds to the equation (2.4) on page 9 in the thesis.
Equation (2.5) in the thesis only denotes the marginal damage per person in
relative terms, otherwise is identical.

**Proposition**
If the marginal product per person $MD_{pp}(n)$ is positive, (i.e. if $MD_{pp}(n) > 0$),
then the production function denoted in equation (1) must have (at least lo-
cally on the interval of $[n, n-1]$) increasing returns to scale. If $MD_{pp}(n) = 0$,
then the production function exhibits constant returns to scale and if $MD_{pp}(n) < 0$,
the production function exhibits decreasing returns to scale.

**Proof**
Suppose a production possibility set:

\[ O = \{(n, y) : y \leq f(n); n \geq a\} \]  

,where:

\[ y = f(n) \]

shows maximum quantity of output $y$ producible from input $n$ and $a$ is the
minimum input for which the production is defined (if there is no minimum,
$a = 0$); $a$ is of course equal to $n - 1$ in our context ($a = n - 1$). At some
specific point $(n, y)$ on this production function, the average productivity $AP$
is:

\[ AP = \frac{f(n)}{n} \]

Locally-increasing returns to scale (RTS) holds at this point if a small
increase in $n$ results in increase in $AP$, locally-decreasing RTS holds at this
point if a small increase in $n$ results in decrease in $AP$ and if the small in-
crease in $n$ doesn’t change $AP$, then we have locally-constant RTS.

In differentiable production function, this means that $\frac{\partial AP}{\partial n} > 0$ for in-
creasing RTS, $\frac{\partial AP}{\partial n} < 0$ for decreasing RTS, and $\frac{\partial AP}{\partial n} = 0$ for constant RTS.
And since $AP = D_{pp}$ as defined in equation (2) and also $MD_{pp}(n) = D_{pp}(n) - D_{pp}(n-1)$ as defined in equation (4), it holds that:

$$\frac{\partial AP}{\partial n} = \frac{\partial D_{pp}}{\partial n} = MD_{pp}(n)$$

(8)

And this means that $MD_{pp}(n) > 0$ for increasing RTS, $MD_{pp}(n) < 0$ for decreasing RTS, and $MD_{pp}(n) = 0$ for constant RTS.

2 Description of inflation-adjustment technique

All damage is inflation-adjusted to the prices of base year 2015 in order to compare the evolution of average damage over time. Such time-evolution is shown in Figure 3.2 on page 16. If the damage would not have been adjusted for inflation, we would see biased results - graphs would tremendously underestimate the damage dealt in the early years of dataset when the prices of goods were significantly lower. The consumption price index (CPI) from the Czech Statistical Office (CSU, 2017) is used (basis index).

Following reference should be added due to the above mentioned amendment:

(1) Czech Statistical Office (2017) [online]. “Oficiální Stránky Českého Statis-

tického Úřadu.” Indexy Spotřebitelských Cen - Inflace - Časové Řady —

Errata

I am deeply sorry for the following errors:

Page 9, line 4
originally is written: "Or in relative terms: $\% \Delta D_{pp}^n = M D_{pp}^n$"
correctly should be written: "Or in relative terms: $\% \Delta D_{pp}^n = % M D_{pp}^n$"

Page 11, line 6
originally is written: $E D_{pp}^n = D_{pp}^n - p_D^n p_{A|R}^n$
correctly should be written: $E D_{pp}^n = D_{pp}^n - p_R^n p_{A|R}^n F$

Page 18, line 14
originally is written: " in Figure A.1"
correctly should be written: " in Figure A.1 in the appendix A"

Page 35, line 5
originally is written: "the probability of detection, $p_D$"
correctly should be written: "the probability of detection, $p_R$"